Beliefs about Others

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Eliciting Beliefs in Games

Why Elicit Beliefs at All?

- Insight into cognition.
- Differentiate between models.
- Uncover patterns.

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Beyond the lab: **'Real-world' examples of beliefs about others:**

Inflation expectations: "Looking ahead, by how much do you expect the prices of your competitors in your main market to change over the next 12 months"

Voting polls: "What percentage of the vote do you think Candidate X will get?"

Asset price expectations: "What do you think will happen to the average price of a home in your postal code over the next 12 months?"

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This lecture: mostly methodological questions: how to do it (and why), what to expect. More on robust patterns next lecture.

Keep in mind: elicit beliefs via another choice.

Issues with choices (noise, mistakes, confusion) also relevant for belief elicitation.

(So far, limited incorporation of this insight to improve elicitation tools.)

Overview

1. Why and How to Elicit Beliefs in Games
2. Effects of Eliciting Beliefs in Games
3. Hedging
4. Best-Responding to Beliefs
5. Second-Order Beliefs
6. Implications and Design Lessons

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Belief Elicitation Techniques

Main Tools:

Non-incentivised elicitation.

Multiple price-list/Random Binary Choice/Becker-DeGroot-Marschak.

Quadratic Scoring Rule.

Binarised (Quadratic) Scoring Rule.

Belief Elicitation Techniques

Non-Incentivised Belief Elicitation

Pros: IC, cheap, fast.

Cons: Not strictly IC, anything goes theoretically;
if cognitively taxing to answer, people tend to go for natural defaults and focal reports;
people may distort answers to justify their actions.

Belief Elicitation Techniques

Becker-DeGroot-Marschak (1964) (BDM)

Random binary choice/multiple price list:
Do you prefer £*X* if *E* or £*X* with probability *p*%?

Vary *p*; choose one question to be paid at random.

Switching point *p* = Belief.

Pros: Strictly IC under monotonicity/statewise-dominance (if constant acts sat. $f(\omega) \succsim g(\omega)\ \forall\omega$, then $f \succsim g$).

Cons: Emphasis on *E* rather than E^C may bias answers (unclear if this was tested, but doesn't tend to show up).

Belief Elicitation Techniques

Brier/Quadratic Scoring Rule (QSR)

- $\max_r \mathbb{E}[a - b(\mathbf{1}_E - r)^2]$.
- Designed in 1950 to incentivise honest reports (leading example being weather reports!).
- Issue: optimal report is biased with non-neutral risk attitudes; some evidence of this exists (Offerman, Sonnemans, van de Kuilen, and Wakker (2009 REStud)).

Belief Elicitation Techniques

Binarised Quadratic Scoring Rule (Hossain and Okui 2013 REStud) (BQSR)

Loss function: $\mathbb{E}_{\theta}[L(\theta, \hat{\theta})]$; estimator $\hat{\theta} \in \arg\min_{\hat{\theta}} \mathbb{E}_{\theta}[L(\theta, \hat{\theta})]$.

Normalise so that $L(\theta, \hat{\theta}) \in [0, 100]$ and pay £X with prob. $L(\theta, \hat{\theta})$ %.

E.g., BQSR:

$$L(E, r) = 100(1 - (1_E - r)^2) \implies \mathbb{E}_{E \sim p}[L(E, r)] \propto p(1 - (1 - r)^2) + (1 - p)(1 - (r)^2)$$

$$\implies p = \arg\min_r \mathbb{E}_{E \sim p}[L(E, r)].$$

Belief Elicitation Techniques

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$$\implies p = \arg\min_r \mathbb{E}_{E \sim p}[L(E, r)].$$

Can do this with any extremum estimator and just binarise it.

Elicit mean, median, variance, etc.

Advantage of binarisation is linearising payment in probabilities (robustness to risk attitudes).

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Can do this with any extremum estimator and just binarise it.

Elicit mean, median, variance, etc.

Advantage of binarisation is linearising payment in probabilities (robustness to risk attitudes).

Pros: Strictly IC under probabilistic sophistication/reduction of subjective-objective compound binary lotteries.

Cons: Complicated to explain, leads to confusion.

Belief Elicitation Techniques

Difficult to Compare

Scoring rule horse race typically using objective probabilities/events.

- Danz, Vesterlund, and Wilson (2022 AER) argue that BQSR leads to misreporting; Healy and Kagel (2025) see low misreporting with both BQSR and BDM.
- BQSR and BDM do similarly well (Burdea and Woon 2022 JEconPsy; Healy and Kagel 2025); higher accuracy than non-incentivised.

Better to explain properly saying something like:

"The payment rule is designed so that you tell us what you believe the probability is that your opponent chooses each action.
To ensure this, your payment is determined as follows: Two numbers are randomly selected between 0 and 100, all being equally likely. You get the bonus if at least one of the two numbers is smaller than the probability you assign to the action chosen by the participant you were randomly matched with."

Overview

Impacts of Elicitation on Behaviour

Eliciting beliefs increases incentivises to form accurate beliefs.

- Does it affect behaviour?

Impacts of Elicitation on Behaviour

Eliciting beliefs increases incentivises to form accurate beliefs.
Does it affect behaviour?

2x2, 2x3, and 3x3 normal-form games: Hoffman (2014), Costa-Gomes and Weizsacker (2008 REStud), Croson (1999 OBHDP; 2000 JEBO).

Also: Nyarko and Schotter (2002 Ecta), Rutstrom and Wilcox (2009 GEB).

Public goods: Croson (2000 JEBO) and Gachter and Renner (2010 EE).

Impacts of Elicitation on Behaviour (Hoffman 2014)

Procedures:

20 normal-form games. Recruit 122 Mannheim undergrad students over 6 sessions, 2012-13.

Baseline Treatment (60): first play all games and then elicit beliefs. Games shown in same order, no feedback.

Belief Treatment (62): elicit choices and beliefs at once, for all games. Games shown in same order, no feedback.

Games always presented from standpoint of row player.

Comprehension questionnaire.

Points = EUR 0.05 Elicit beliefs using QSR (up to EUR 2.00).

3 actions and 3 beliefs for different games randomly selected for payment.

Avg earnings EUR 15; duration up to 90min.

Impacts of Elicitation on Behaviour (Hoffman 2014)

Table 1: Game Properties

Overview of Properties for all Games

Characteristic / Game #	1	2	3	4	5	7	9	11	13	15	17	19
unique NE		X	X	X	X	X	X	X		X	X	X
NE Pareto dominated	(X)	X	X		X	X				X		
symmetric game	X	X	X	X								
3x3	X		X	X		X	X	X	X			
3x2										X	X	X
2x2		X			X							
dominance solvable	X	X	X		X	X			X	X	X	
elimination of weakly dom. strategy	X								X			
column dominated action	X	X	X			X		X	X		X	
row dominated action	X	X	X		X					X		
symmetric alternative	-	-	-	-			X		(X)			X

Notes: Game numbers given in the table correspond to each game in the appendix. Of course there are symmetric alternatives in symmetric games. (X) for NE Pareto dominated means that the other outcome is also a NE, (X) for symmetric alternative means that no player is better off with the symmetric alternative compared to the NE.

Games:

Games 2x2, 3x3, 2x3, with at least one PSNE.

4 symmetric + 8 asymmetric x row/column roles (adding/subtracting constant, random ordering stractegies).

Impacts on Choices and Beliefs (Hoffman 2014)

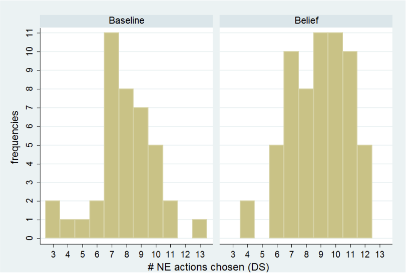

Figure 1: Histogram of NE actions for both treatments

Impact on Choices:

Freq. NE strat. play: 54.6% $\rightarrow$ 58% (not significant)

Freq. NE strat. play in dominance-solvable games: 61% $\rightarrow$ 68% (p-val<.01)

Freq. non-rationalisable play decreases but (not econ. nor stat. significant).

Freq. dominated action decreases (22.2% $\rightarrow$ 15.6%).

Impacts on Choices and Beliefs (Hoffman 2014)

Impact on Beliefs:

- Not significantly different (although tests are based on a somewhat arbitrary partition rather than the actual answers.)
- Best-response rate increases from about 50% to about 60%.

Impacts on Choice Sophistication (Hoffman 2014)

Table 2: Percentage of actions matched by models' predictions

Treatment	Games	NE	L1	L2	L3	D1	D2	Ef	Maxmin
Baseline	not DS	0.43	0.57	0.48	0.43	0.53	0.41	0.47	0.51
	DS	0.61	0.60	0.59	0.50	0.56	0.52	0.41	0.58
	All	0.55	0.59	0.55	0.47	0.55	0.48	0.43	0.56
Belief	not DS	0.39	0.58	0.50	0.40	0.55	0.44	0.50	0.57
	DS	0.68	0.60	0.65	0.56	0.61	0.59	0.33	0.62
	All	0.58	0.59	0.60	0.51	0.59	0.54	0.39	0.60
Average	**All**	**0.56**	**0.59**	**0.58**	**0.49**	**0.57**	**0.51**	**0.41**	**0.58**

Table 3: Percentage of subjects classified by models' actions most taken - only dominant solvable games

	Clear Cases		Ties		Clear cases and ties	
Model	Baseline	Belief	Baseline	Belief	Baseline	Belief
NE	13.33	29.03	3.33	1.61	16.66	30.64
L1	13.33	14.52	5.00	1.61	18.33	16.13
L2	11.67	12.90	5.00	1.61	16.67	14.51
L3	0.00	0.00	0.83	0.00	0.83	0.00
D1	1.67	3.23	4.17	1.61	5.84	4.84
D2	0.00	0.00	0.00	0.81	0.00	0.81
Ef	15.00	4.84	2.50	0.81	17.50	5.65
Maxmin	15.00	14.52	2.50	1.61	17.50	16.13
not identified	30.00	20.97	6.67	11.29	6.67	11.29

Notes: The second and third column only states unique predictions. The ties, which are reported in the fourth and fifth column, are ties between exactly two models. In case of a tie the subjects is classified as 50% for each model.

Choices:

Level-*k* classification suggest higher belief treatment consistent with higher sophistication

Impacts on Behaviour (Costa-Gomes and Weizsacker, 2008 REStud)

Procedures:

7 3x3 asymmentric normal-form games + 7 games adding/subtracting payoffs. All have ! PSNE, 5 out 7 are dominance solvable.

Vary order of elicitation: Actions and 1st-order beliefs:
Treatments: A1 (40), 1A (42), 1A1A (46), 1AQ (47), and 1A1AC (44).
A1: all actions first, then all beliefs; 1A: opposite;
1A1A: beliefs then actions game by game;
Q: quiz about QSR; C: calculator available.

219 students Boston area.

Pay one; point = USD 0.15; QSR for beliefs. No feedback. Play once each game.

Impacts on Behaviour (Costa-Gomes and Weizsacker, 2008 REStud)

TABLE 1

Games classified by strategic structure and models' predicted actions

Game	Dominance solvable	Rounds of dominance	Nash	Naïve L 1	L2	D1	Optimistic
#1	Y	2,3	T-L	M-L	T-M	T-L	B-M
#2	Y	3,2	M-L	M-M	T-L	M-L	T-R
#3	Y	2,3	B-R	T-M	B-M	B-M	M-M
#4	Y	3,2	M-M	T-L	T-M	T-M	T-R
#5	Y	2,3	T-M	B-L	T-L	T-L	M-L
#6	Y	3,2	B-M	M-R	M-M	M-M	M-L
#7	Y	2,3	M-R	B-R	M-R	M-R	T-M
#8	Y	3,2	B-R	B-L	B-R	B-R	T-M
#9	Y	3,4	T-R	T-L	M-R	T-M	M-L
#10	Y	4,3	B-L	T-L	B-M	M-L	T-M
#11	N	–,–	M-M	B-M	M-R	B-M	T-L
#12	N	–,–	B-L	M-R	M-M	M-R	T-L
#13	N	–,–	T-R	T-M	B-R	T-M	M-L
#14	N	–,–	T-L	B-M	M-M	B-M	T-R

Procedures:

7 3x3 asymmentric normal-form games + 7 games adding/subtracting payoffs. All have ! PSNE, 5 out 7 are dominance solvable.

Impacts on Behaviour (Costa-Gomes and Weizsacker, 2008 REStud)

#1	L	M	R
T	78, 73	69, 23	12, 14
M	67, 52	59, 61	78, 53
B	16, 76	65, 87	94, 79

#2	L	M	R
T	21, 67	59, 57	85, 63
M	71, 76	50, 65	74, 14
B	12, 10	51, 76	77, 92

Game #2's pay-offs are obtained by subtracting 2 points from Game #1's pay-offs.

#3	L	M	R
T	74, 38	78, 71	46, 43
M	96, 12	10, 89	57, 25
B	15, 51	83, 18	69, 62

#4	L	M	R
T	73, 80	20, 85	91, 12
M	45, 48	64, 71	27, 59
B	40, 76	53, 17	14, 98

Game #4's pay-offs are obtained by adding 2 points to Game #3's pay-offs.

#5	L	M	R
T	78, 49	60, 68	27, 35
M	10, 82	49, 10	98, 38
B	69, 64	42, 39	85, 56

#6	L	M	R
T	39, 99	36, 28	57, 86
M	83, 11	50, 79	65, 70
B	11, 50	69, 61	40, 43

Game #6's pay-offs are obtained by adding 1 point to Game #5's pay-offs.

Procedures:

7 3x3 asymmentric normal-form games + 7 games adding/subtracting payoffs. All have ! PSNE, 5 out 7 are dominance solvable.

Impacts on Behaviour (Costa-Gomes and Weizsacker, 2008 REStud)

Limited Effects of Belief Elicitation?

Fisher exact tests to check for differences pair by pair of games and treatments. 9 out of 280 comparisons with p-values under 5%. Categorised beliefs as well.

NE play: A1 (35.8%), 1A (32.6%), 1A1A (37.9%), 1AQ (37.0%), and 1A1AC (38.6%). (60% of choices coincide with L1 and 40% with L2.)

Authors conclude that “treatment effects on play are minor”.

Given (1) amount of noise (NE play close to uniform) and (2) small sample size, unclear if this is a robust conclusion.

Impacts on Behaviour in DS 2x2 Games (Croson 1999 OBHDP; 2000 JEBO)

Procedures:

5 different 2x2 games; 10 rounds.

80 Uni of Pittsburgh students; quiz; different opponents each period.

Row/Col players' action labels A,B/1,2.

Points = money, avg earnings USD 10.02 + USD 3 show-up fee; duration 1h.

Treatments: 'Control' without belief elicitation vs. 'Guess' with belief elicitation.
Same (fixed) order of games.

Unclear, but seems like belief elicitation not incentivised.

Impacts on Behaviour in DS 2x2 Games (Croson 1999 OBHDP; 2000 JEBO)

TABLE 1

Shafir and Tversky's Prisoners' Dilemma

	Cooperate	Defect
Cooperate	75,75	25,85
Defect	85,25	30,30

TABLE 2

Asymmetric Prisoners' Dilemma

	Cooperate	Defect
Cooperate	85,65	35,75
Defect	95,15	40,20

TABLE 3A

Iterated Dominance 1

	1	2
A	85,75	25,90
B	50,25	60,40

TABLE 3B

Iterated Dominance 2

	1	2
A	75,85	25,50
B	90,25	40,60

TABLE 4

Same Reason

	1	2
A	30,30	25,85
B	85,25	75,75

Impacts on Behaviour in DS 2x2 Games (Croson 1999 OBHDP; 2000 JEBO)

Effects of Belief Elicitation:

		Control	Beliefs
	Sym Prisoner’s Dilemma	77.5	55.0
Freq. Dominated play:	Asym Prisoner’s Dilemma	62.5	42.5
	Iterated Dominace	40.0	20.0
	Prisoner’s Delight	12.5	5.0

Iterated Dominance Play:

BR to opponent’s dominant strat.: 35% (control) $\rightarrow$ 55% (beliefs).

Impacts on Behaviour in DS 2x2 Games (Croson 1999 OBHDP; 2000 JEBO)

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Iterated Dominance Play:

BR to opponent's dominant strat.: 35% (control) $\rightarrow$ 55% (beliefs).

Beliefs predict choice:

Sym PD: Pr(cooperate | guess cooperate) = 83%; Pr(cooperate | guess defect) = 32%.

Asym PD: Pr(cooperate | guess cooperate) = 53%; Pr(cooperate | guess defect) = 35%.

(Note: monetary payoffs = actual payoffs.)

Impacts on Behaviour in AMP Games (Rutstrom and Wilcox 2009 GEB)

Procedures:

- Hide-seak AMPx, col hiding symmetric 1/0 payoffs; row seaker asymmetric x if match on up, 1 if match on down, 0 ow; $x = 19$.
- 264 participants; 36 periods, fixed pairs.
- Treatments: 80 pax No Beliefs; 92 pax (unincentivised) Expected Choice guess; 92 pax (Quadratic) Scoring Rule.

Impacts on Behaviour in AMP Games (Rutstrom and Wilcox 2009 GEB)

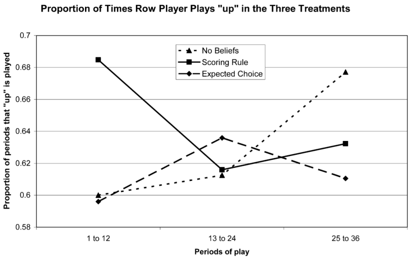


Fig. 1. Proportion of times row player plays "up" in the three treatments.

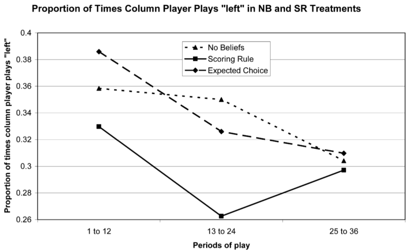


Fig. 2. Proportion of times column player plays "left" in the three treatments.

Effects of Belief Elicitation:

Eliciting beliefs affects choices.

(Remainder of the paper estimates structural models.)

Impacts on Behaviour in Public Goods Games (Croson 2000 JEBO)

Procedures: Experiment 1.

Linear public goods. $u_i = P \sum_j s_j + k(E_i - s_i)$, $s_i \in [\mathbf{0}, E_i]$.

$k/|I| < P < k$: dominant strategy is contributing zero.

48 participants; fixed groups of 6.

2 rounds with 10 periods each; feedback each period.

Points = money, avg earnings USD 14.30 + USD 5 show-up fee; duration 1h.

Treatments: 'Control' without belief elicitation vs. 'Guess' with belief elicitation.

Strange scoring rule: guess total contribution of others; get 0.50 if correct, 0.25/abs. distance.

Impacts on Behaviour in Public Goods Games (Croson 2000 JEBO)

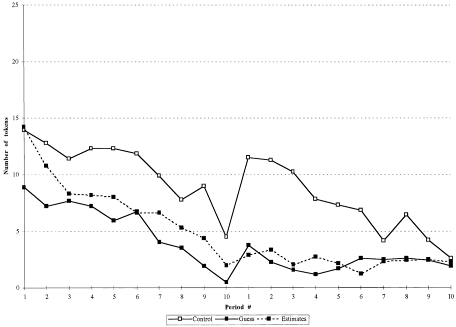


Fig. 1. Average contributions and estimates.

Table 1
Average tokens in public good

	Round									
	1	2	3	4	5	6	7	8	9	10
Control	13.96	12.83	11.42	12.33	12.33	11.88	9.92	7.79	9.04	4.54
Guess	8.92	7.25	7.71	7.25	6.00	6.79	4.08	3.58	1.96	0.54
Difference	5.04*	5.58*	3.71*	5.08*	6.33*	5.08*	5.83*	4.21	7.08*	4.00*
Restart control	11.54	11.33	10.29	7.88	7.33	6.88	4.21	6.50	4.25	2.67
Restart guess	3.79	2.29	1.58	1.21	1.71	2.63	2.54	2.67	2.50	1.96
Difference	7.75**	9.04**	8.71**	6.67**	5.63**	4.25**	1.67*	3.83**	1.75*	0.71

* $p < 0.05$; ** $p < 0.01$.

Effects of Belief Elicitation: lower contributions.

Impacts on Behaviour in Public Goods Games (Gachter and Renner 2010 EE)

Procedures: Experiment 1.

Linear public goods. $u_i = P \sum_j s_i + k(E_i - s_i)$, $s_i \in [\mathbf{0}, E_i]$.

$k/|I| < P < k$: dominant strategy is contributing zero.

204 participants students at Erfurt and Nottingham; groups of 4.

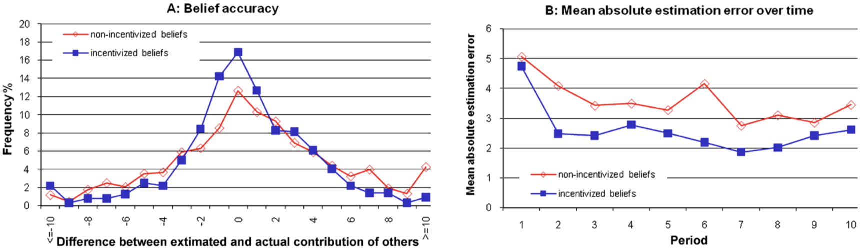
1 round with 10 periods each; feedback each period.

Points = money, avg earnings USD 14.30 + USD 5 show-up fee; duration 1h.

Treatments: 72 'No beliefs'; 68 'Non-incentivised beliefs'; 65 'incentivised beliefs'.

Same strange scoring rule: guess total contribution of others; get 0.50 if correct, 0.25/abs. distance.

Impacts on Choices and Beliefs (Gachter and Renner 2010 EE)



Effects of Incentivising Belief Elicitation: more accurate beliefs; effect only significant with learning.

Impacts on Behaviour in Public Goods Games (Gachter and Renner 2010 EE)

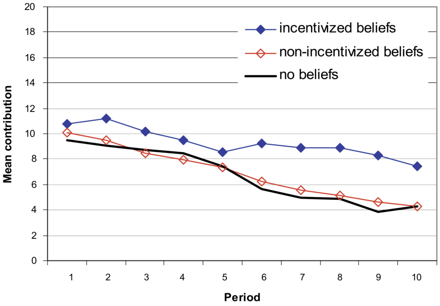


Fig. 3 Contribution levels over time

Effects of Belief Elicitation: *higher* contributions.

Impacts of Elicitation on Behaviour

Eliciting beliefs *can* affect behaviour.

Not a settled question!

- Effect seems to depend on games: e.g., in DS games tends to lead to choices consistent with higher level of k-rationalisability.
- Unclear if/how much paying for beliefs matters in general.
- Arguably effect less noticeable when games are less straightforward.

Important for Experimental Design:

- If want to maximise action-belief consistency, elicit choices and beliefs at the same time (more later).
- Unclear if/how this interacts with other treatments, i.e., affects comparative statics.

Overview

Hedging

Example:

Consider coordination game, 1 if match, 0 if ow.

Pay x for choice and x for belief accuracy (e.g., with BSR).

Hedging: choose A, guess B wp1, guarantee oneself x for sure.

Risk attitudes bias answers.

Blanco, Engelmann, Koch, Normann (2010 EE) study whether hedging is actually an issue.

Hedging (Blanco et al. 2010 EE)

Table 1 Overview of experimental treatments

Sequential prisoners' dilemma (SPD) experiment [Sect. 4]		Coordination game experiment [Sect. 5]	
Royal Holloway, Univ. of London, UK (October 2007–March 2008)		LEE, Copenhagen University, Denmark (November 2008–March 2009)	
SPDHedge	$N = 30$	Hedge	$N = 40$
SPDNoHedge	$N = 30$	NoHedge	$N = 26$
		NoHedgeStrong	$N = 48$
		BinaryHedge (binary belief)[a]	$N = 26$
		BinaryNoHedge (binary belief)[a]	$N = 28$
		BinaryNoHedgeStrong (binary belief)[a]	$N = 26$
		SafeHedge (binary belief, safe option)[a]	$N = 28$

Total number of observations: 282

[a]The online appendix reports details on these treatments

Experiment 1 (Blanco et al. 2010 EE)

Procedures:

Sequential Prisoner's Dilemma:

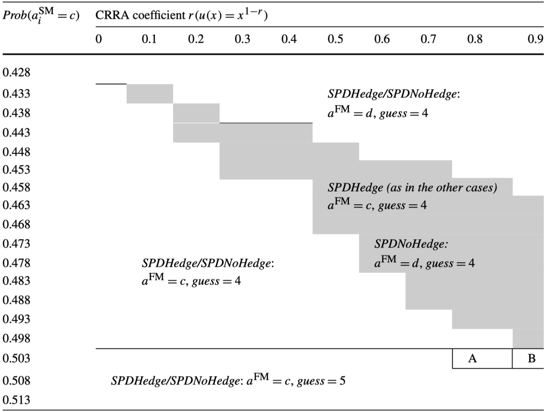
(1) 2nd-mover choice; (2) QSR guess task prob. cooperate; (3) 1st-mover choice.

Hedge: pay both; NoHedge: pay one chosen at random (double amounts).

No feedback. 30 pairs.

Experiment 1 (Blanco et al. 2010 EE)

Table 3 Best responses in *SPDHedge* and *SPDNoHedge*



A: *SPDHedge*: $a^{FM} = c$, $guess = 4$/*SPDNoHedge*: $a^{FM} = c$, $guess = 5$

B: *SPDHedge*: $a^{FM} = c$, $guess = 4$/*SPDNoHedge*: $a^{FM} = d$, $guess = 5$

Experiment 1 (Blanco et al. 2010 EE)

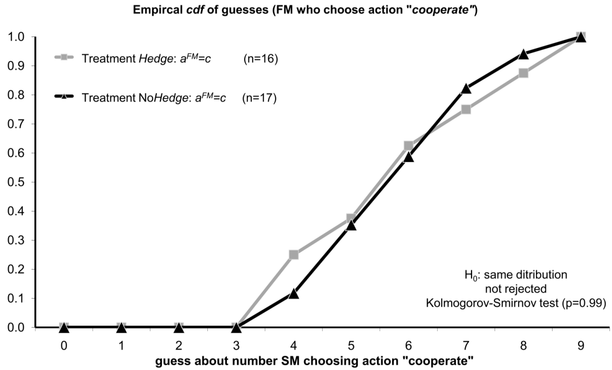


Fig. 2 Beliefs about SM play (stated by FM cooperators)

Outcomes: no impact.

Experiment 2 (Blanco et al. 2010 EE)

Table 4 Payoff table for the coordination game

Player *i*	Player *k*	
	X	*Y*
X	(0,0)	(16,14)
Y	(14,16)	(0,0)

Procedures:

Coordination:

- 40 pax Hedge: pay both; 26 pax NoHedge: pay one chosen at random; 48 pax NoHedgeStrong: paragraph stressing no hedging.
- No feedback. Points = money. Random matching.
- Avoid symmetric 50-50 outcome.

Experiment 2 (Blanco et al. 2010 EE)

Table 5 Payoffs for the guess task in the coordination game treatments *Hedge* and *NoHedge*

Stated belief	Actual choice of matched player k	
	X	Y
Rather X than Y		
$x5$: strongly X	15	0
$x4$	13	1
$x3$	11	2
$x2$	9	3
$x1$: weakly X	7	4
Rather Y than X		
$y1$: weakly Y	4	7
$y2$	3	9
$y3$	2	11
$y4$	1	13
$y5$: strongly Y	0	15

Procedures:

Elicit categorical beliefs.

Hedging: $(X, x.)$ and $(Y, y.)$.

Experiment 2 (Blanco et al. 2010 EE)

Table 6 Overview of results in the coordination game experiment

	Hedge	*NoHedge*	*NoHedgeStrong*	*NoHedge* (pooled)
N	40	26	48	74
I. $(X, x\cdot)$ or $(Y, y\cdot)$[a]	16	9	17	26
(%)	40.00%	34.62%	35.42%	35.14%
II. $(X, x5)$ or $(Y, y5)$ ["(pseudo-)hedgers"]	13	4	8	12
(%)	32.50%	15.38%	16.67%	16.22%
III. Hedging possibility stated[b]	15	2	0	2
(%)	37.50%	7.69%	0.00%	2.70%
IV. $(X, x5)$ or $(Y, y5)$ and hedging poss. stated	11	0	0	0
(%)	27.50%	0.00%	0.00%	0.00%
II. as % of $(X, x\cdot)$ or $(Y, y\cdot)$ choices	81.25%	44.44%	47.06%	46.15%
IV. as % of $(X, x\cdot)$ or $(Y, y\cdot)$ choices	68.75%	0.00%	0.00%	0.00%

[a] $x\cdot$ stands for stated belief $x1$, $x2$, $x3$, $x4$, or $x5$, and $y\cdot$ for stated belief $y1$, $y2$, $y3$, $y4$, or $y5$

[b] The subject explains in the non-structured post-experimental questionnaire that there is a hedging opportunity

Results: Evidence for hedging. Massively underpowered.

Some subjects hedge when they should not: may be helpful to explain subjects when they should not hedge.

Explicit mentions of hedging in comments. Debrief questionnaires valuable.

Experiment 2 (Blanco et al. 2010 EE)

Table 7 Test for differences between *Hedge* and *NoHedge*

	Hedge		*NoHedge*	*NoHedgeStrong*	*NoHedge* (pooled)
"(Pseudo-)hedgers"[a]	13	vs	4	8	12
"Non-hedgers"[b]	27		22	40	62
Fisher exact test (two-sided p-value)			0.155	0.131	0.058
Boschloo test (two-sided p-value)			0.125	0.112	0.048
Hedging possibility stated[c]	15	vs	2	0	2
Not stated	25		24	48	72
Fisher exact test (two-sided p-value)			0.009	<0.001	<0.001
Boschloo test (two-sided p-value)			0.006	<0.001	<0.001

[a] $(X, x5)$ or $(Y, y5)$ choices

[b] All other choices

[c] The subject explains in the non-structured post-experimental questionnaire that there is a hedging opportunity

Results: Evidence for hedging. Massively underpowered.

Design lesson: Avoid hedging!

Overview

Best-Responding to Beliefs

Do people actually best-respond to stated beliefs?

Best-Responding to Beliefs (Costa-Gomes and Weizsacker, 2008 REStud)

Beliefs:

- Text input: about 2/3 of beliefs in multiples of 10pp; 90% in multiples of 5pp.
- Beliefs tend to track opponents' actions but biased toward uniform.
- Overestimate prob. dominated actions: avg belief 15.7%; freq. 10.4%; 34.7% assign prob. zero.

Best-Responding to Beliefs (Costa-Gomes and Weizsacker, 2008 REStud)

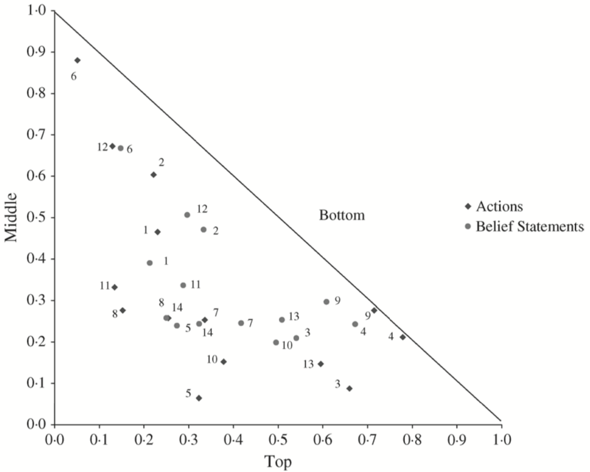


FIGURE 3

Action frequencies and average belief statements (pooled across treatments and isomorphic player roles, numbering according to Row player's perspective)

Beliefs tend to track action freqs., but less extreme.

Best-Responding to Beliefs (Costa-Gomes and Weizsacker, 2008 REStud)

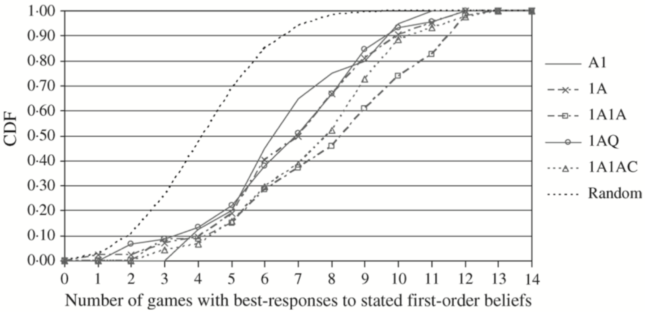


FIGURE 4

(a) Empirical p.d.f. and (b) empirical c.d.f. of number of subjects with *x* best-responses to stated first-order beliefs

Best-Responses to Beliefs:

BR freq.: A1 (50.6%), 1A (52.6%), 1A1A (60.0%), 1AQ (51.4%), and 1A1AC (57.1%). Authors state "Exact two-sample Kolmogorov-Smirnov tests, pairing the five treatments in all possible ways, yield no p-value less than 5%."

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Based on structural exercise,

results suggest that subjects play games as if attributing a low degree of response rationality to their opponents – as if they expected the opponents to play randomly. But in contrast, when subjects state beliefs they ascribe to their opponents the ability to choose actions that are best responses to beliefs, which, in turn, seem to be uniform over the player's own decisions.

My interpretation: 1A1A mitigates this disconnect and increases action-belief consistency, but a properly powered test is necessary.

Beliefs and Actions (Rey-Biel 2009 GEB)

Rey-Biel (2009 GEB): 3x3 with unique PSNE.

- 10 constant sum (CS), 10 similar variable sum (VS) games.
- Games: Allow for equal split of payoffs (F) by not choosing NE; or not (U).
- One vs Two digits in payoffs (multiply by 7).
- Play 10 games, 8 dominance solvable.
- 2004, 2006; ELSE/ELFE! 160 participants; 80 per role. Quiz.
- Random order of games, anonymous matching, pay one, no feedback.
- Points = money.
- A1: first all actions, then all beliefs. QSR for paying beliefs.

Best-Responding to Beliefs (Rey-Biel 2009 GEB)

Table 2
Percentages of equilibrium actions and best-responses by treatments

% of equilibrium actions

Row		1D F	1D U		2D F	2D U
	CS	78	89	CS	74	74
	VS	58	74	VS	65	58

Column		1D F	1D U		2D F	2D U
	CS	84	76	CS	71	81
	VS	55	64	VS	66	55

% of best-responses

Row		1D F	1D U		2D F	2D U
	CS	65	70	CS	64	75
	VS	65	64	VS	67	65

Column		1D F	1D U		2D F	2D U
	CS	64	67	CS	83	67
	VS	57	61	VS	74	66

NE:

More in: 1D than 2D; CS than VS.

Best-responding to beliefs:

CS than VS; 2D than 1D; unclear F vs U.

Beliefs and Actions (Rey-Biel 2009 GEB)

Table 3
Percentage of actions matched by models' predictions

Treatment	Subject	*NE*	*L1*	*L2*	*L3*	*D1*	*MM*	*Eq*	*Mm*	*mM*	*Ef*
CS	Row	79	42	68	70	65	20	47	50	37	25
	Column	78	54	69	69	73	30	41	47	46	29
	Average	**78.4**	**48**	**68.5**	**69.5**	**69**	**25**	**44**	**48.5**	**41.5**	**27**
VS	Row	64	56	52	58	66	37	36	49	42	38
	Column	60	56	65	60	68	41	43	62	37	42
	Average	**61.9**	**56**	**58.5**	**59**	**67**	**39**	**39.5**	**55.5**	**39.5**	**40**

Table 5
Average frequency of stated beliefs on models' predictions

Treatment	Subject	*NE*	*L1*	*L2*	*L3*	*D1*	*MM*	*Eq*	*Mm*	*mM*	*Ef*
CS	Row	55	50	53	51	55	39	39	43	36	30
	Column	59	47	51	52	57	33	39	44	44	29
	Average	**57**	**48.5**	**52**	**51.5**	**56**	**36**	**39**	**42**	**40**	**29.5**
VS	Row	49	50	48	49	51	44	36	51	33	40
	Column	48	50	41	44	52	44	36	43	45	40
	Average	**48.5**	**50**	**44.5**	**46.5**	**51.5**	**44**	**36**	**47**	**39**	**40**

Also: NE much much better predictor in CS than VS games.

Overview

1. Why and How to Elicit Beliefs in Games
2. Effects of Eliciting Beliefs in Games
3. Hedging
4. Best-Responding to Beliefs
5. Second-Order Beliefs
6. Implications and Design Lessons

Second-Order Beliefs and Actions

Sherlock Holmes, pursued by his opponent, Moriarty, leaves London for Dover. The train stops at a station on the way, and he alights there rather than traveling on to Dover.

He has seen Moriarty at the railway station, recognizes that he is very clever and expects that Moriarty will take a faster special train in order to catch him in Dover. [...]

But what if Moriarty had been still more clever and had foreseen his actions accordingly? Then, obviously, he would have traveled to the intermediate station. Holmes, again, would have had to calculate that and he himself would have decided to go on to Dover.

Whereupon, Moriarty would again have reacted differently.

Morgenstern (1928)

Second-Order Beliefs and Actions

Why do we care about 2nd-order beliefs?

Contagion arguments, iterative reasoning.

Psychological games, where intentions matter and payoffs depend on beliefs, e.g., guilt-avoidance (Charness and Dufwenberg 2006 Ecta; Vanberg 2008 Ecta).

Second-Order Beliefs and Actions (Manski and Neri 2013 GEB)

Manski and Neri (2013 GEB) elicit 1st and 2nd order beliefs in binary hide-and-seek game (Matching pennies).

Hider's action: $\sigma_H \in \{0, 1\}$;
Hider's 1st-order beliefs about Seeker $\hat{\sigma}_S \in [0, 1]$;
Hider's 2nd-order beliefs, $\mu_H \in \Delta([0, 1])$.

Weak Coherence: $F_{\mu_H}(1/2) > 1/2 \iff \hat{\sigma}_S > 1/2$.

Strong Coherence: $F_{\mu_H}(1/2) = 1 - \hat{\sigma}_S$.

x**%-Strong Coherence:** $|F_{\mu_H}(1/2) - (1 - \hat{\sigma}_S)| \leq x/100$.

Best-Response: $\sigma_H = 1/2 - 1/2 \cdot 1\{\hat{\sigma}_S > 1/2\} + 1/2 \cdot 1\{\hat{\sigma}_S < 1/2\}$.

Procedures:

Northwestern students. Duration 30min.

USD 10.00 if win.

Discretise support of 2nd-order beliefs $\hat{\sigma}_{-i} \in \{[0, .05], (.05, .20], (.20, .50], (.50, .80], (.80, .95], (.95, 1.00]\}$

QSR for both 1st and 2nd order beliefs.

Treatments A12, A21, 1A2, 12A, 21A, 2A1.

Treatments (Manski and Neri 2013 GEB)

Table 4
Participants and treatments.

Session name	Number of participants	Number of rounds	Number of observations	Task order
C-1-2	20	4	80	choice, 1st-order beliefs, 2nd-order beliefs
C-2-1	16	4	64	choice, 2nd-order beliefs, 1st-order beliefs
1-C-2	18	4	72	1st-order beliefs, choice, 2nd-order beliefs
1-2-C	16	4	64	1st-order beliefs, 2nd-order beliefs, choice
2-C-1	26	4	104	2nd-order beliefs, choice, 1st-order beliefs
2-1-C	18	4	72	2nd-order beliefs, 1st-order beliefs, choice
All	114	4	456	

Actions (Manski and Neri 2013 GEB)

Table 5
Frequency of choice A, choice outcome, and occurrence of Seeker's wins. $(X, Y) = $ (Hider's choice, Seeker's choice).

	Period				
	1 (%)	2 (%)	3 (%)	4 (%)	All (%)
Choice A among Hiders	60	60	61	60	60
Choice A among Seekers	35	39	37	54	41
Choice outcome					
(A, A)	18	23	26	32	25
(A, B)	42	37	35	28	36
(B, A)	18	16	11	23	17
(B, B)	23	25	28	18	23
Seeker wins	40	47	54	49	48

Table 6
Frequency of 1st-order beliefs $P = 0.50$ among Hiders and among Seekers in the pooled dataset.

Frequency of 1st-order beliefs $P = 0.5$	Period				
	1 (%)	2 (%)	3 (%)	4 (%)	All (%)
Hiders	39	53	53	49	48
Seekers	49	51	39	40	45
Any role	44	52	46	45	46

Second-Order Beliefs and Actions (Manski and Neri 2013 GEB)

Table 10
Summary statistics, over the pooled dataset, of $Q(0.5)$: the probability that a subject assigns to her opponent's first-order beliefs being not higher than 50 percent.

No	456
min	0
mean	0.45
10th percentile	0
25th percentile	0.35
median	0.50
75th percentile	0.50
90th percentile	0.70
max	1

Second-Order Beliefs and Actions (Manski and Neri 2013 GEB)

Table 14

Relationship between choice, 1st- and 2nd-order beliefs, according to whether: (i) the observed choice is consistent with optimal response to 1st-order beliefs, (ii) the observed choice is consistent with optimal response to 2nd-order beliefs, (iii) 1st- and 2nd-order beliefs are coherent (according to 0%-, 5%-, 10%-*strong coherence* or *weak coherence*), (i)–(iii) hold simultaneously. Panel 1 presents the *empirical* percentage frequencies of observations for which conditions (i), (ii), and/or (iii) hold. Panel 2 presents the *theoretical* percentage probabilities of observations for which conditions (i), (ii), and/or (iii) *would* hold, computed under the assumption that participants' answers (choice, 1st- and 2nd-order beliefs) are random. The first row of each panel reports frequencies computed over the entire sample, while the remaining rows divide the data in subsamples according to whether 1st- and/or 2nd-order beliefs imply or not indifference.

	(i) Choice and 1st-order beliefs	(ii) Choice and 2nd-order beliefs	(iii) 1st- and 2nd-order beliefs				(i)–(iii) Choice, 1st- and 2nd-order beliefs				Obs.	
			Strong coherence			Weak coherence	Strong coherence			Weak coherence		
			0%	5%	10%		0%	5%	10%			
Panel 1: Empirical percentage frequencies	%	%	%	%	%	%	%	%	%	%	No.	%
All obs.	89	75	34	40	52	83	33	38	46	68	456	100
$P \neq 0.5,\ Q(0.5) \neq 0.5$	81	57	19	28	37	55	17	25	32	47	173	38
$P \neq 0.5,\ Q(0.5) = 0.5$	79	100	0	10	39	100	0	10	32	79	71	16
$P = 0.5,\ Q(0.5) \neq 0.5$	100	59	0	9	26	100	0	5	12	59	92	20
$P = 0.5,\ Q(0.5) = 0.5$	100	100	100	100	100	100	100	100	100	100	120	26
Panel 2: Theoretical percentage probabilities assuming participants' answers to be random	%	%	%	%	%	%	%	%	%	%	No.	%
All obs.	73	71	26	34	41	81	26	29	32	54	456	100
$P \neq 0.5,\ Q(0.5) \neq 0.5$	50	50	0	9.75	19	50	0	2.4375	4.75	25	173	38
$P \neq 0.5,\ Q(0.5) = 0.5$	50	100	0	10	20	100	0	5	10	50	71	16
$P = 0.5,\ Q(0.5) \neq 0.5$	100	50	0	10	20	100	0	5	10	50	92	20
$P = 0.5,\ Q(0.5) = 0.5$	100	100	100	100	100	100	100	100	100	100	120	26

BR freq. high, but 1st/2nd-order belief consistency close to random.

Too complicated elicitation of 2nd-order beliefs?

- Binarised version of 2nd-order elicitation could have been more successful.

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Key Takeaways

1. Incentivised belief elicitation may affect behaviour and people may exploit hedging opportunities.
2. Choices and stated beliefs largely but not perfectly consistent.
3. Beliefs tend to track opponents' actions, but uniform biased.
4. People seem to have higher-order uncertainty about opponents' beliefs.

Revisiting Questions

Are choices explained by beliefs about others?

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- What if we tell people about others' action frequencies?

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- What about strategic setting itself? I.e., own/other payoffs to actions?
  What kinds of patterns should we expect to be robust?
  Tend to track opponents, but more next lecture.
- Relatedly: Do people understand equilibrium effects? 24 Feb presentation.

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Beliefs about what?

- Not just 1st-order uncertainty; also 2nd-order uncertainty.
- People seem to have higher-order uncertainty about opponents' beliefs. Limited
  evidence on what people actually form beliefs about.
  Opponent sophistication matters. Opponent assessment of outcomes?
  Opponent reaction to incentives? Opponent reasoning?

Anything else comes to mind?

Design Decisions for Eliciting Beliefs about Others

Some questions for discussion:

What to elicit: probabilities, odds?

How to represent: numbers, figures?

Wording: ‘chance’, ‘likelihood’, ‘probability’? ‘uniform’ or ‘equally likely’?
Simplify when possible without compromising accuracy.

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How to elicit: sliders, input boxes?

- Sliders more natural, but induce motor bias and centre/compromise bias.
- Input boxes likely to mitigate centre bias, but require numerical sophistication, lead to mass points of multiples of 5/10, and pose an adding-up-to-100 problem with non-binary events.

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Hedging: Pay beliefs or actions, not both!
Otherwise, prone to hedging/bias due to risk/uncertainty attitudes.

Pay one/some/all: it depends;
pay one seems to work at least as well as pay all;
pay all makes incentives clear but requires independent tasks (ow hedging) and more prone to 'income effects';
good rationale for some is unclear.

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More questions?

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Shouldn’t small details *not* matter?

Design Decisions for Eliciting Beliefs about Others

Shouldn't small details *not* matter? Yes and no.

Comparative statics ought to be robust to fine details
(insofar as instructions are theoretically equivalent).

But fine details may affect noise in data.

Point estimates are always likely to be affected (by noise);
e.g., measures of choice consistency.

Design Decisions for Eliciting Beliefs about Others

Shouldn't small details *not* matter? Yes and no.

- Comparative statics ought to be robust to fine details (insofar as instructions are theoretically equivalent).
- But fine details may affect noise in data.
- Point estimates are always likely to be affected (by noise); e.g., measures of choice consistency.

Useful surveys:

- Schotter and Trevino (2014 ARE) Belief Elicitation in the Laboratory.
- Schlag, Tremewan, and van der Weele (2015 EE) A penny for your thoughts: a survey of methods for eliciting beliefs.
- Charness, Gneezy, and Rasocha (2021 JEBO) Experimental methods: Eliciting beliefs.
- Healy and Leo (2025 Ch) Belief Elicitation: A User's Guide.